CARMA Correlators

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CARMA Correlators Overview

- System Block Diagram
- Terminology and Technology
  - Spectra
  - Downconversion
  - Sideband separation
  - Delay tracking
  - Correlation
  - Continuum (Wideband) and Spectral modes
  - FPGAs
- Correlator Systems
  - 8-antenna wideband system
  - 15-antenna spectral line system
    - 15-antenna dual-polarization mode
    - 23-antenna single-polarization mode
- System Architecture
  - How it all works together
- Correlator Room Tour
Correlator Operation

Some magic occurs ...
CARMA Antenna Signal Path

- 15-antennas
- 8-bands
CARMA Antenna Signal Path

Antenna

\[ \phi_1 = \text{phase-switch and lobe-rotation correction} \]

~100GHz or ~270GHz + \phi_1

Sky spectrum

1GHz to 9GHz Receiver Output

Block Downconverter

1GHz to 5GHz Receiver Output

10GHz

5GHz to 9GHz Receiver Output

Downconverter

1GHz to 5GHz

Band selection

0.5GHz to 1GHz Downconverter Output

Digitizer

1GHz clock

8-bit ADC and 1:2 Demux

500MHz clock, 2 x 8-bits, real-valued

To digitizer board FPGAs
CARMA Antenna Signal Path

From digitizer

Digitizer interface FPGA

1:4 Demux

125MHz clock, 8 x 8-bits, real-valued

Whole nanosecond delay

Sub-nanosecond delay

Digitizer filter FPGA

250MHz + $\phi_2$

$\phi_2$ = secondary lobe-rotation correction

Digital filtering and decimation

Modulation to real-values $\frac{B}{2}$

$B = 500\text{MHz}, 250\text{MHz}, 125\text{MHz}, \ldots, 1.95\text{MHz}$

Requantization

180-degree phase-switch removal

Autocorrelation

Front-panel LVDS transmitters (to cross-correlator boards)
Antenna Signal Path Terminology

- Spectra

- Complex numbers
  - Complex-conjugate
  - Complex-valued exponential (phasor)
  - Hermitian

- Downconversion
  - Upper and Lower sidebands

- Sideband separation
  - (Walsh) Phase-switching

- Delay tracking
  - Phase-offset tracking (lobe rotation)
  - Phase-slope tracking
Spectra

- Provide information on signal power as a function of frequency
  - Auto-spectra
    - Magnitude (power) only
    - Dominated by antenna noise component; used for diagnostics
  - Cross-spectra
    - Magnitude (power) and phase (relative time or location)
    - Radio source information used for imaging
  - Continuum (wideband) sources
    - Constant power over a wide range of frequencies
  - Spectral-line sources
    - Power concentrated in a narrow range of frequencies

- Fourier Transform
  - Decomposes a signal into cosines and sines (complex exponentials) of different frequencies

- Fourier Theorems and Pairs
  - Auto-correlation \iff\ Auto-spectra
  - Cross-correlation \iff\ Cross-spectra
The Complex Plane

- Cartesian format;
  - \( c = a + j b \)
  - \( c^* = a - j b \) (complex conjugate)
- Polar format;
  - \( c = r \{ \cos(\theta) + j \sin(\theta) \} = r \exp(j \theta) \)
  - \( c^* = r \{ \cos(\theta) - j \sin(\theta) \} = r \exp(-j \theta) \)
Fourier Transform

\[ X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft) dt \]

\[ X[k] = \sum_{n=0}^{N-1} x[n] \exp\left(-j \frac{2\pi n}{N} k\right) \]

- Represents signal samples \( x[n] \) in terms of cosine and sinusoid functions
- Provides an alternative view of a signal (and its components)
- Efficient computation via the Fast Fourier Transform
Discrete Fourier Transform

Signal

$x[n]$

Sample Index, $n$

Discrete Fourier Transform

$X[k]$

Frequency Channel, $k$

Real Fourier Components

$\cos\left(\frac{2\pi n}{N} \times 2\right)$

$\cos\left(\frac{2\pi n}{N} \times 4\right)$

$\cos\left(\frac{2\pi n}{N} \times 6\right)$

Sample Index, $n$

Amplitude

The DFT was normalized by $N/2$ (the expected peak of a cosine)
## Fourier Theorems

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Time</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time shift</td>
<td>$x(t - t_0)$</td>
<td>$X(f) \exp(-j2\pi ft_0)$</td>
</tr>
<tr>
<td>Frequency shift</td>
<td>$x(t) \exp(j2\pi f_0 t)$</td>
<td>$X(f - f_0)$</td>
</tr>
<tr>
<td>Time-domain convolution (Frequency-domain multiplication)</td>
<td>$x(t) * y(t)$</td>
<td>$X(f)Y(f)$</td>
</tr>
<tr>
<td>Time-domain multiplication (Frequency-domain convolution)</td>
<td>$x(t)y(t)$</td>
<td>$X(f) * Y(f)$</td>
</tr>
<tr>
<td>Time-domain correlation</td>
<td>$x(t) * y(t)$</td>
<td>$X(f)Y^*(f)$</td>
</tr>
</tbody>
</table>
Complex-valued Exponentials

- Time-varying complex-valued signal

- Frequency-shift Fourier theorem:
  - Modulation = Up-conversion
    \[ x(t) \exp(+j2\pi f_0 t) \Leftrightarrow X(f - f_0) \]
  - Demodulation = Down-conversion
    \[ x(t) \exp(-j2\pi f_0 t) \Leftrightarrow X(f + f_0) \]

- Euler’s Formula (Polar format with \( \theta = 2\pi f_0 t \)) :
  \[
  \begin{align*}
  \exp(+j2\pi f_0 t) &= \cos(2\pi f_0 t) + j \sin(2\pi f_0 t) \\
  \exp(-j2\pi f_0 t) &= \cos(2\pi f_0 t) - j \sin(2\pi f_0 t) \\
  \cos(2\pi f_0 t) &= \frac{1}{2} \{ \exp(-j2\pi f_0 t) + \exp(+j2\pi f_0 t) \} \\
  \sin(2\pi f_0 t) &= \frac{j}{2} \{ \exp(-j2\pi f_0 t) - \exp(+j2\pi f_0 t) \}
  \end{align*}
  \]
Complex-valued Exponentials

\[ \exp(+j2\pi f_0 t) \]

Complex exponential = a convenient way to manipulate a cosine and a sine

Real-valued cosine and sine signals consist of counter-rotating exponential phasors

Real
In-phase
Even
Symmetric

Imaginary
Quadrature
Odd
Anti-symmetric
### Fourier Pairs

<table>
<thead>
<tr>
<th>Time</th>
<th>Frequency</th>
<th>Spectra</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\exp(+j2\pi f_0 t)$</td>
<td>$\delta(f - f_0)$</td>
<td><img src="null" alt="Graph" /></td>
</tr>
<tr>
<td>$\exp(-j2\pi f_0 t)$</td>
<td>$\delta(f + f_0)$</td>
<td><img src="null" alt="Graph" /></td>
</tr>
<tr>
<td>$\cos(2\pi f_0 t)$</td>
<td>$\frac{1}{2} {\delta(f + f_0) + \delta(f - f_0)}$</td>
<td><img src="null" alt="Graph" /></td>
</tr>
<tr>
<td>$\sin(2\pi f_0 t)$</td>
<td>$\frac{i}{2} {\delta(f + f_0) - \delta(f - f_0)}$</td>
<td><img src="null" alt="Graph" /></td>
</tr>
</tbody>
</table>

Real-valued spectra contain complex-conjugate components = Hermitian
Demodulation

- Most signal processing systems use an LO with both in-phase and quadrature components and two mixers.

- The CARMA telescopes use a single mixer and time-multiplex the LO between in-phase and quadrature:
  - 90-degree phase switching
Demodulation and Lobe Rotation

Relative arrival delay causes a phase-slope

Phase-offset added to first LO

Phase-offset Tracking = Lobe Rotation

\[ \phi_{LR1} = -j 2 \pi f_{LO1} \tau \]
Sideband Separation

In-phase

\[ I(f) \]

Quadrature

\[ Q(f) \]

\[ jQ(f) \]

Sideband separation = 90-degree phase-switch demodulation

\[
U(f) = \frac{1}{2} \{ I(f) + jQ(f) \} \quad L(f) = \frac{1}{2} \{ I(f) - jQ(f) \}^* 
\]
Phase-switched Noise Source

In-phase

\[ I(f) \]

Quadrature

\[ Q(f) \]

The noise source is sideband separated to appear in the LSB
Lobe Rotation

- **Time-shift Fourier theorem:**
  \[ x(t - \tau) \Leftrightarrow X(f) \exp(-j2\pi f \tau) \]

- **Primary lobe rotation:**
  - Generated by the lobe rotator and applied to the first LO

- **Secondary lobe rotation:**
  - Applied by the correlator
Correlation

Some magic occurs ...
Cross-correlation (full-precision)

\[ \mathcal{N}(0, \rho) \quad \mathcal{N}(0, 1-\rho) \]

\[ \times \quad \times \quad = \quad = \]

\[ \mu = \rho \quad \sigma^2 = 2\rho^2 \]
\[ \mu = 0 \quad \sigma^2 = (1-\rho)^2 \]
\[ \mu = 0 \quad \sigma^2 = \rho(1-\rho) \]
\[ \mu = 0 \quad \sigma^2 = \rho(1-\rho) \]

K-sample average:

\[ \mathcal{N}\left(\rho, \frac{1+\rho^2}{K}\right) \]
Correlation

- **Lag (XF) correlator**
  - Measure the averaged product of pairs of antenna signals over a range of time delays
  - Fourier transform to get cross-power spectrum

- **FX correlator**
  - Fourier transform or digitally filter antenna-based signals into frequency-domain channels
  - Multiply and average in the frequency domain

- **CARMA uses lag correlators**
Lag Correlation

- Lag calculation:

\[ r_{xy}[n] = \frac{1}{K} \sum_{k=0}^{K-1} x[k]y[k - n] \quad n = -\frac{N}{2} \text{ to } \left(\frac{N}{2} - 1\right) \]

- Spectrum calculation (via DFT/FFT):

\[ R_{xy}[m] = \sum_{n=0}^{N-1} r_{xy}[n] \exp\left[-\frac{j2\pi mn}{N}\right] \quad m = 0 \text{ to } \frac{N}{2} \]
Lag Correlation

Cross-correlations are calculated for lags $-N/2$ to $N/2-1$

Auto-correlations are calculated for lags 0 to $N/2$
2-bit Sampling (Re-quantization)

- **Thresholds:**
  - The input amplitudes (values) at which output codes (quantization) decisions are made
  - Eg., whether an input value becomes 00b or 01b
  - The input amplitude is determined by the downconverters
  - **Threshold optimization** maximizes the correlator efficiency
2-bit Correlation

- 2-bit sampling (or filter output requantization)
- 2-bit by 2-bit multiplication
  - 4-input lookup table (LUT)
  - Nominal 4-bit multiplication result
  - Deleted inner product has 3-bit result (25% logic reduction for 1% loss in SNR)
  - Correlation estimate $\mathcal{N}(\rho, 1.31/K)$
  - 87% efficiency relative to full-precision
Cross-correlation Spectra

Magnitude measures image pixel strength

Phase measures image pixel location (relative to pointing center)
Continuum vs Spectral-line Mode

Data taken: 11/24/2009

500MHz continuum (3C454)

500MHz line (Upsilon Herculis SiO maser)

62MHz line
Correlator Systems

- **Wideband (SZA)**
  - 8-antenna, 28-baselines, 8GHz bandwidth
  - 16 bands; fixed IF location
  - 500MHz bandwidth per band

- **Spectral-line (CARMA)**
  - 15-antenna, 105-baselines, 4GHz bandwidth
  - 8 bands; independently tuneable over the IF
  - 2MHz to 500MHz bandwidth per band
Correlator Systems

CARMA Correlator
3 bands x 15-telescopes x adjustable LO x
500MHz down to 2MHz bandwidth

SZA Correlator
16 bands x 8-telescopes x fixed
500MHz bandwidth
CARMA Digitizer/Correlator Board
CARMA Digitizer/Correlator Board

- 2-antennas input
- 500MHz bandwidth each input
- FIR filtering
- Digital Downconversion
- Digital Delay
- Auto/Cross Correlation

- DATA-FPGAs average correlation data
- SYS-FPGA coordinates real-time DMA of averaged data to PowerPC memory
- PowerPC runs Linux
- Processes the data in ‘sustained real-time’, i.e., it just has to keep up!

FPGA = Field Programmable Gate Array
COBRA Digitizer Board

- 2-antennas input
- 500MHz bandwidth each input
- FIR filtering (62MHz and lower)
- Digital Downconversion
- Digital Delay (whole-sample only)
- Auto/Cross Correlation

- DATA-FPGAs average correlation data
- SYS-FPGA coordinates real-time DMA of averaged data to DSP memory
- DSP runs uC/OS-II real-time OS
- DATA-FPGAs have 20K logic elements (versus 360K on CARMA digitizer)

Serial-to-parallel (2-bits@1GHz to 32-bits@62.5MHz)

1GHz PLL

1GHz 6-bit ADC

33MHz DSP and System Controller FPGA (under the digitizer module)

Data Processing FPGAs
COBRA Correlator Board

- Calculates 10 cross-correlations
- SYS-FPGA coordinates real-time DMA of averaged data to DSP memory
- DSP runs uC/OS-II real-time OS
- DATA-FPGAs have 50K logic elements (versus 520K on CARMA digitizer)
CARMA Dual-band Modes

- 4 bands
  - Independently tuneable over the IF
  - 2MHz to 500MHz bandwidth per band

- 15-antenna dual-polarization
  - 15-antenna, 105-baselines, 4-polarizations, 2GHz bandwidth

- 23-antenna single-polarization
  - 23-antenna, 253-baselines, 2GHz bandwidth
Single-polarization Mode

(8-antenna hypothetical ‘CARMA’ system)
Dual-polarization Mode

Digital Data:
2-antennas x 2-polarizations x 2-bits x 500MHz
Single-polarization Dual-band Mode
System Architecture

- FPGA data processing;
  - Digital filtering
  - Delay tracking
  - Correlation (and accumulation to 1/64 sec)
  - 180-degree phase-switch demodulation

- On-board processor performs;
  - Control and monitoring
  - Data processing
    - Fourier transformation
    - 90-degree phase-switch demodulation
    - Accumulation (1/2 sec)

- Crate processor performs;
  - Control
  - Band data and monitor point collection
  - CARMA control system interface

- CARMA Control System, Pipeline, and Archive
  - Commands to individual bands
  - Monitor points to real-time displays
  - Pipeline and archive process data from multiple bands
That’s it for correlator ‘magic’

Time for a tour of the correlator room