Bima memo #20

BIMA

Technical Memo #1

RMS Deformation of Antenna #3 Using Structural Analysis

by W. L. Urry

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Abstract

Structural analysis of the Hat Creek interferometer third antenna indicates that RMS surface error due to the flexure of the back-up structure under the forces of gravity is only .00061 inch. The considerable stiffness of the structure means that the panel surface alignment to an accuracy of .002 inch at an elevation of 5° should be preserved at all elevation angles. Finite element analysis aided by the use of Zernike circle polynomials makes it possible to separate the RMS contributions due to tilt and focus from other surface deformations. The Zernike circle polynomials also enable us to obtain a complete quantitative description of the deformed surface.

Discussion

The finite element analysis program used in this study, SAP86, is a microprocessor variant of the standard SAP IV program developed at U.C. Berkeley. The study of a complex structure begins with the computer entry of a large list of nodes which define the beam, truss or plate connectivity of the structure. The result of the analysis is a corresponding list of node displacements that may be expected in the structure under the given load conditions. In general these node displacements may be displayed graphically. The resulting image is often too complex to grasp easily, and since the deformation is three dimensional, several views of the deformed structure are often required. If two structures are to be compared, then the graphical approach is completely inadequate.

The RMS deformation of an antenna surface under load is a useful measure of antenna performance since it is a direct measure of aperture efficiency. A simple quantitative measure of antenna performance is to calculate the RMS node movement of the surface nodes under its loading extremes. The relative weight of each node, in the calculation, must be proportional to the amount of panel area it supports. The RMS node movement, so obtained, is easy to determine and it is useful for the first order comparison of structures. It does have a disadvantage in that it is overly pessimistic. It includes as part of the RMS error any error due to tilt. Tilt does not contribute to wavefront cancellation, but is instead a pointing error which may be removed with a pointing calibration. The weighted RMS node movement also includes any error due to a shift in focus. Focus shift also does not contribute to wavefront cancellation if the antenna can be refocused as it is moved. Although it is desirable to know the magnitude of the focus shift in order to keep it within limits it does not, in general, degrade antenna performance. The removal of tilt and focus shift from the weighted RMS node movement requires fitting a plane to the node displacement data to remove tilt and the fitting of a parabola to remove focus shift. This fitting is conveniently accomplished with the aid of Zernike polynomials.

The circle polynomials of Zernike are used in the field of optics. They are a set of functions which are defined on the unit circle with a magnitude of 1 at unit radius. They are orthogonal and complete. Any reasonably behaved function within the unit circle may be expressed as a collection of Zernike polynomials. As may be expected from a set of functions which are orthogonal and complete, Parsevals' theorem applies. The overall RMS deviation of the surface is equal to the square root of the sum of the squares of the properly weighted Zernike coefficients describing the surface. An attractive feature of the Zernike assay is the fact that the circle polynomials correspond to distortions which are familiar to the optician. The first few coefficients describe the degree of tilt, focus shift, astigmatism, third order coma and third order spherical aberration. The Zernike assay can provide a complete quantitative description of the surface. Contour maps of the Zernike polynomials provide detailed information about surface deformation and point the way to correct any structural defect.

We carried out a Zernike assay of the Hat Creek interferometer third antenna back-up structure. Using SAP86, we determined the node movement between the pointing extremes of horizon to vertical in order to estimate the maximum RMS distortion we can expect under our current observing conditions. We also modeled the node movement which occurred in both directions from an angle of 45 degrees elevation. The tables on page 4 show the Zernike coefficients in units of inches for the node movement resulting from each of these conditions. The total RMS is shown along with the result of excluding the tilt as well as the result of excluding both tilt and focus. Evidently, most of the node movement is due to tilt, and removing the tilt contribution eliminates most of the deviation. These numbers must be compared with the present panel alignment accuracy of .002 inch RMS. Moving the antenna from horizontal to vertical produces a surface error due to node movement of only .00088 inch RMS after removing tilt. If focus shift is also removed then the surface error becomes .00061 inch RMS. The errors due to flexure are negligible compared to alignment errors. The three figures following page 4 show the node deflections graphically for each of the angles considered.

The panel alignment to an accuracy of .002 inch was carried out by radio holography at an elevation angle of 5°. Structural analysis of the antenna back-up structure indicates that this figure should be well maintained at all elevation angles.

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Ref:

Born and Wolf, *Principals of Optics.* Pergamon Press 1975 Appendix VII

Michael Budiansky, "An Atlas of Zernike Functions," Ten Meter Telescope internal note #34, 23 March 1982.

Dick Plambeck, "Antenna 3 Efficiency Measurements," Radio Astronomy Laboratory Technical Report #101, 31 March 1986.

Zernike Functions

n	m	function	rms	
1	1	$\rho cos\theta$	.500	tilt
2	0	$2\rho^2 - 1$	.577	focus shift
2	2	$\rho^2 cos2\theta$	.408	astigmatism
3	1	$(3\rho^3 - 2\rho)cos\theta$	.353	third order coma
3	3	$\rho^3 cos3\theta$	.353	
4	0	$6\rho^4 - 6\rho^2 + 1$	.447	third order spherical aberration
4	2	$(4\rho^4 - 3\rho^2)cos2\theta$	.316	
4	4	$\rho^4 cos4\theta$	.316	
5	1	$(10\rho^5 - 12\rho^3 + 3\rho)cos\theta$	.289	
5	3	$(5\rho^5 - 4\rho^3)cos3\theta$	.289	
5	5	$\rho^5 cos5\theta$	.289	
6	0	$20\rho^6 - 30\rho^4 + 12\rho^2 - 1$	.378	
6	2	$(15\rho^6 - 20\rho^4 + 6\rho^2)cos2\theta$	.267	
6	4	$(6\rho^6 - 5\rho^4)cos4\theta$	.267	
6	6	$\rho^6 cos6\theta$	.267	

$$rms = \sqrt{\frac{1}{2(n+1)}} \text{ for } m \neq 0$$
$$rms = \sqrt{\frac{1}{n+1}} \text{ for } m = 0$$

rms represents the number that must multiply the Zernike coeficient in order to turn it into the rms contribution of the Zernike function.

Zernike Coefficients for Horizon to Vertical

n m	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
0							0.0012						
1						0.0000		0.0066					
2					0.0000		0.0011		0.0001				
3				0.0000		0.0000		0.0015		0.0002			
4			0.0000		-0.0000		-0.0002		0.0000		-0.0001		
5		0.0000		-0.0000		-0.0000		-0.0009		-0.0000		-0.0001	
6	0.0000		-0.0001		-0.0001		-0.0002		0.0000		0.0000		0.0000

RMS Error for Horizon to Vertical

RMS, excluding const. term - - - - - - - - - - - - -	0.00342 inch
RMS, exculding tilt (A11) - - - - - - - - - - - - -	0.00088 inch
RMS, excluding tilt, focus (A11, A20) - - - - - - -	0.00061 inch

Zernike Coefficients for Vertical to 45 Degrees

n m	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
0							0.0003						
1						0.0000		0.0046					
2					0.0000		0.0003		0.0000				
3				0.0000		0.0000		0.0010		0.0001			
4			0.0000		-0.0000		-0.0001		0.0000		-0.0000		
5		0.0000		-0.0000		-0.0000		-0.0006		-0.0000		-0.0001	
6	0.0000		-0.0000		-0.0000		-0.0001		0.0000		0.0000		-0.0000

RMS Error for Vertical to 45 Degrees

RMS, excluding const. term - - - - - - - - - - - - -	0.00234 inch
RMS, exculding tilt (A11) - - - - - - - - - - - - -	0.00044 inch
RMS, excluding tilt, focus (A11, A20) - - - - - - -	0.00040 inch

Zernike Coefficients for 45 Degrees to Horizon

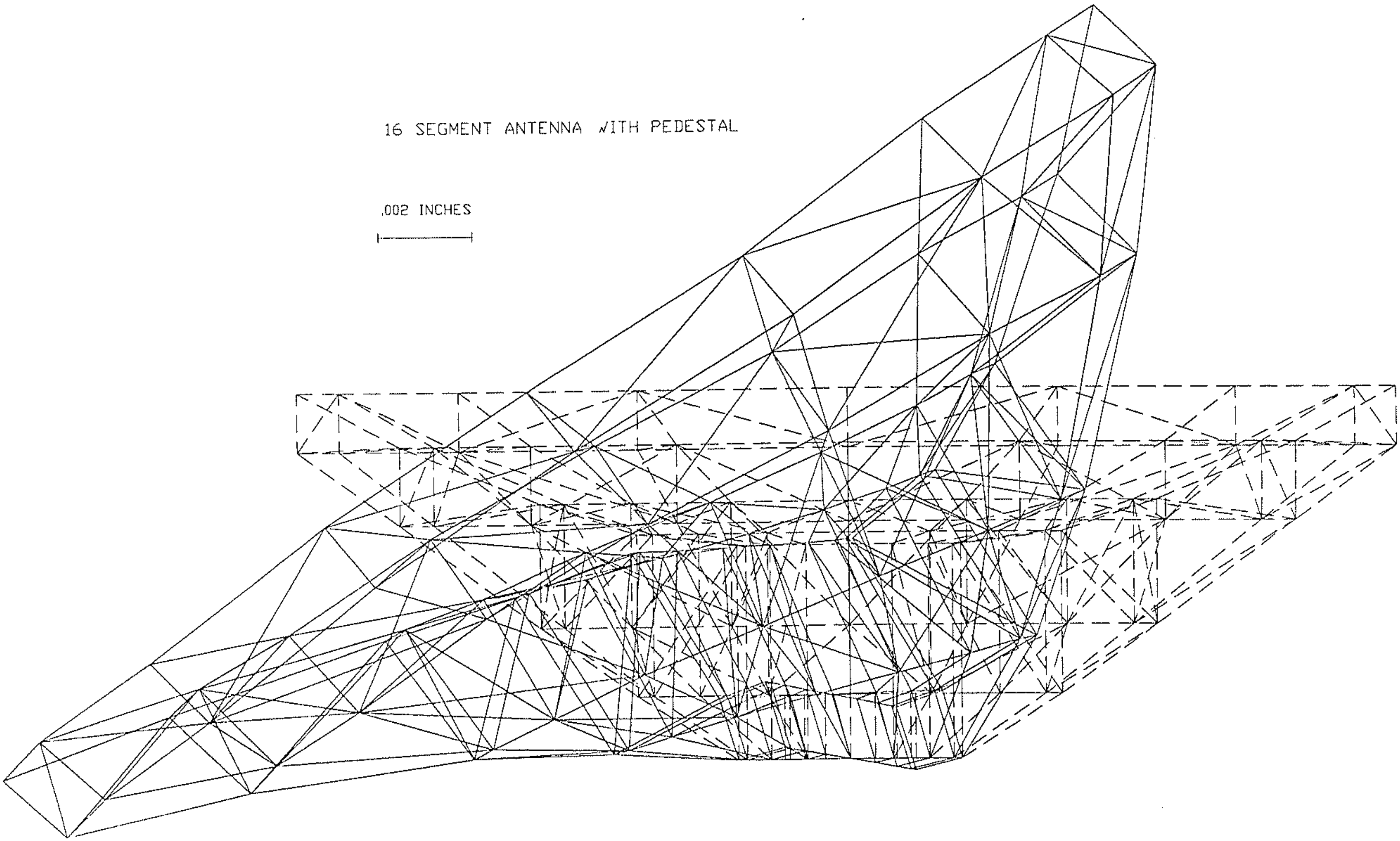
n m	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
0							-0.0008						
1						0.0000		-0.0019					
2					-0.0000		-0.0008		-0.0000				
3				0.0000		0.0000		-0.0004		-0.0001			
4			0.0000		0.0000		0.0002		0.0000		0.0001		
5		-0.0000		-0.0000		-0.0000		0.0003		0.0000		0.0000	
6	-0.0000		-0.0000		-0.0000		0.0002		0.0000		-0.0000		-0.0000

RMS Error for 45 Degrees to Horizon

RMS, excluding const. term - - - - - - - - - - - - -	0.00108 inch
RMS, exculding tilt (A11) - - - - - - - - - - - - -	0.00051 inch
RMS, excluding tilt, focus (A11, A20) - - - - - - -	0.00021 inch

16 SEGMENT ANTENNA WITH PEDESTAL

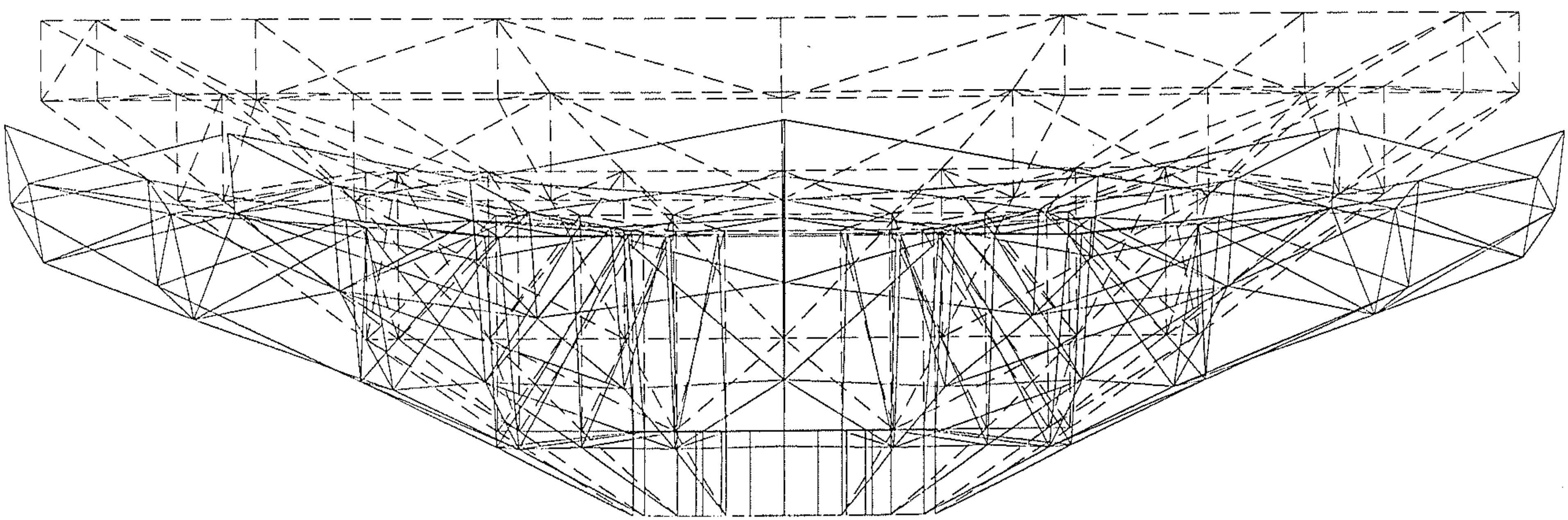
.002 INCHES



16 SEGMENT ANTENNA WITH PEDESTAL

DNANT 10

.002 INCHES



16 SEGMENT ANTENNA WITH PEDESTAL
AT 45 DEGREE ORIENTATION

.002 INCHES