Berkeley Illinois Maryland Association

Memo

Subject Ray-tracing the BIMA Reflectors
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Abstract

A study has been made of the BIMA Cassegrain optics to assess alignment tolerances, depth of focus, and aberrations, and the present degree of correction of the system. This has been done both by ray-tracing the design specification for the Cassegrain system, and also by investigating holograms made by correlating one antenna with another. The surface accuracy was assessed by RMS and efficiency computations. Residual aberrations are expressed as coefficients of Zernike polynomials, and these can be used to characterize the system.

1. Introduction

The inauguration of the millimeter receivers has placed new demands on the telescope optics, and made it desirable to take another look at alignment tolerances and surface accuracy.

Alignment tolerances are easily explored by tracing rays through the optical system, where various tilts and offsets can easily be introduced, and their effect observed in changes of the optical path lengths of the rays. A convenient arrangement is to situate a point object at the feed phase center, and trace rays “backwards” to a reference plane in front of the primary reflector.

The author developed a set of programs for the VAX computer some time ago, collectively named "Oski," in honor of a local figure on campus. Essential portions of Oski have now been ported to Fortran 77 on our Sun Unix system, and given a somewhat friendlier face (written in C). Some new programs were added for doing Zernike wavefront analysis. These have been extended to work with holograms made at HCO as well.

2. The BIMA Reflector Optics

The following parameters were assumed for the BIMA reflector optics.

Y_1	Semi-diameter of Primary Mirror	120 inches
Y_2	Semi-diameter of Secondary Mirror	12 "
F_1	Focal Length of Primary Mirror	100.8 "
F_{21}	1st Focal Length of Secondary Mirror	9.6 "
F_{22}	2nd Focal Length of Secondary Mirror	109.04 "
B	Back Focal Length (behind primary)	18 "
m	Amplification Ratio, F_{22} / F_{22}	11.17
EFL	Equivalent Focal Length of System	1126.15 inches
f	Focal Ratio of System	4.69
R_p	Petzval Radius of Focal Surface	12.01 inches
h	Feed Horn Offset	2.6 "

The primary is parabolic (conic constant $K = -1$), and the secondary hyperbolic.

When the antenna is directed to the test transmitter for holography, the finite distance of the transmitter (~200,000 inches) requires a focus adjustment backward of 6.5 inches, or equivalently, movement of the secondary *away* from the primary by 0.07 inch. Holography may be conducted at the infinity focus setting without difficulty, however; a wavefront curvature adjustment has to be made to the phases.

The depth of focus may conservatively be taken as $f^2\lambda$, which works out to be 2.6 inches at 3mm, but only 0.86 inch at 1 mm. This is still greater than the deviation of our Petzval surface from a plane. The Petzval surface is defined to be the surface of sharpest focus if the optical system is free of aberrations, but is approximately the locus of sharpest focus in our case, with only mild coma present.

3. A Word about Zernike Polynomials

The classic reference for this subject is Born and Wolf (1975), section 9.2. Another is Murison (1995). Budiansky (1982) is worth looking up in the Keck Telescope Technical Note series.

The Zernike polynomials are orthogonal functions over the unit circle. They take the form

$$Z_n^m(\rho,\theta) = R_n^m(\rho)\, e^{jm\theta}, \tag{1}$$

where the radial functions, $R_n^m(\rho)$, are given by:

$$R_n^m(\rho) = \sum_{s=0}^{\frac{n-m}{2}} (-1)^s \frac{(n-s)!}{s!(\frac{n-m}{2}-s)!(\frac{n+m}{2}-s)!} \rho^{n-2s}. \tag{2}$$

The first few radial polynomials are:

n \ m	0	1	2	3	4	5	6
0	1						
1		ρ					
2	$2\rho^2 - 1$		ρ^2				
3		$3\rho^3 - 2\rho$		ρ^3			
4	$6\rho^4 - 6\rho^2 + 1$		$4\rho^4 - 3\rho^2$		ρ^4		
5		$10\rho^5 - 12\rho^3 + 3\rho$		$5\rho^5 - 4\rho^3$		ρ^5	
6	$20\rho^6 - 30\rho^4 + 12\rho^2 - 1$		$15\rho^6 - 20\rho^4 + 6\rho^2$		$6\rho^6 - 5\rho^4$		ρ^6

The orthogonality relations for Z and R are:

$$\int_0^1 \int_0^{2\pi} Z_n^m(\rho,\theta) Z_{n'}^{m'}(\rho,\theta)\rho d\rho d\theta = \frac{\pi}{n+1}\delta_{nn'}\delta_{mm'}, \tag{3}$$

and

$$\int_0^1 R_n^m(\rho) R_{n'}^{m'}(\rho)\rho d\rho = \frac{1}{2}\frac{1}{n+1}\delta_{nn'}. \tag{4}$$

It is useful to introduce a real function $U_m^n(\rho,\theta)$ defined by:

$$U_n^m(\rho,\theta) = \begin{cases} R_n^m(\rho)\cos m\theta, & m \geq 0 \\ R_n^{|m|}(\rho)\sin m\theta, & m < 0 \end{cases}. \tag{5}$$

This is done, since, in general, the wavefront deformation, Φ, is a real function of the pupil coordinates. Φ is simply the path length through the optical system along ray trajectories, typically traced from the focus back to the entrance pupil of the system. In practice, Φ is taken to be the difference in path length between that of a given ray, and the principal ray traced through the center of the system. Since the wavefront deformations (aberrations) vary for the most part with small integral powers of ρ, and depend on small factors of the azimuthal coordinate θ, we can expand Φ with just a few Zernike functions. In general, we can write

$$\Phi(\rho,\theta) = \sum_{n=0}^{\infty} \sum_{\substack{m=-n \\ \text{by } 2}}^{n} A_n^m U_n^m(\rho,\theta). \tag{6}$$

To obtain the A_n^m coefficients, we make use of the above orthogonality conditions to obtain

$$A_n^m = K(m)\,\frac{n+1}{\pi} \int_0^{2\pi}\!\!\int_0^1 \Phi(\rho,\theta) U_n^m(\rho,\theta)\rho d\rho d\theta, \tag{7}$$

where

$$K(m) = \begin{cases} 2, & m \neq 0 \\ 1, & m = 0 \end{cases}.$$

This factor results from the orthogonality condition on cos($m\theta$).

The more common aberrations have Zernike functions associated with them:

$A_1^1 \rho \cos\theta$	Tilt
$A_2^0 (2\rho^2 - 1)$	Defocus
$A_2^2 \rho^2 \cos 2\theta$	Astigmatism
$A_3^1 (3\rho^3 - 2\rho)\cos\theta$	Coma
$A_4^0 (6\rho^4 - 6\rho^2 + 1)$	Spherical Aberration

(Not shown for the terms with cosines are their -m partners with the same radial component but sines in place of cosines.)

The difference between these and the Seidel representations is that the Zernike polynomials frequently carry with them lower-order terms. Thus, spherical aberration carries with it some defocus and a phase retardation (the latter not regarded as an aberration). It turns out happily that the amounts of lower-order aberration introduced are just right for maximizing the intensity at the center of the reference sphere for the phases.

The Parseval's formula for Φ is:

$$\overline{\Phi^2} = \sum_{n=0}^{\infty} \frac{{A_n^0}^2}{n+1} + \tfrac{1}{2} \sum_{n=1}^{\infty} \sum_{\substack{m=-n \\ m\neq 0, \text{by } 2}}^{n} \frac{{A_n^m}^2}{n+1}. \tag{8}$$

The intensity at some point P, formed by our wavefront at the focus, divided by the intensity, arising from an aberration-free wavefront, is called the Strehl ratio, or Strehl intensity. It is given exactly by

$$i(P) = \frac{1}{\pi^2}\left|\int_0^{2\pi}\int_0^1 \exp\left[\frac{2\pi j}{\lambda}\Phi(P)\right]\rho d\rho d\theta\right|^2 . \tag{9}$$

The Strehl ratio is a close cousin of the radio astronomers' "aperture efficiency." It neglects such items as spillover. For small aberrations, the Strehl ratio is approximately

$$i(P) \approx 1-\left(\frac{2\pi}{\lambda}\right)^2\left[\overline{\Phi^2(P)}-\left[\overline{\Phi(P)}\right]^2\right] = 1-\left(\frac{2\pi}{\lambda}\right)^2(\Delta\Phi)^2 \tag{10}$$

The utility of (8) is then apparent, telling us that the RMS ($\Delta\Phi$) for each mode is simply $A_n^m / \sqrt{2(n+1)}$, or $A_n^0 / \sqrt{n+1}$, and (10) then enables us to estimate the penalty.

4. The Data

4.1. Ray-Tracing

The technique used to ray-trace the reflectors was to place a point object at the feed position, which was assumed to lie 2.6 inches above the optical axis in, let us say, the *y*-direction, with no displacement in the *x*-direction. This results in approximately a 0.133 degree tilt of the wavefront after rays are traced through the optics. Accordingly, a tilted plane was placed in front of the primary reflector. Radial fans of rays were generated at evenly spaced azimuth angles and traced through the system. For the various test cases, the tilt of the final aperture plane was adjusted slightly to remove most of the residual tilt in the ray data prior to Zernike analysis. The Oski ray-trace program generates optical path distance data in addition to (*x,y,z*) coordinate data for the final ray crossings. A "filter" program was required to translate these data into ρ,θ,Φ, and appropriate weights for the Zernike analysis. It is customary to represent Φ and A_n^m in units of λ, or turns of phase. That has been done in all data presented in this report.

In the various data sets shown below, each has as its top line an over-all Strehl loss, computed directly from equation (9). It includes "benign" residual tilt and defocus, and tends to be pessimistic. The next 7 lines are Zernike coefficients, each line for a different n. For each line, the various *m* modes are shown. These are the A_n^m values, computed from equation (7). The next 7 lines are corresponding power losses (deductions from Strehl) based on the approximation (10). When the Zernike coefficients are greater in magnitude than, say, 0.05, these should be taken with a grain of salt, albeit they are shown in their relative importance. Commonly, these losses will add up to a greater sum than the first line. Caution is then required. From ray-tracing, noise can enter the computations via lost rays, sampling artifacts, etc.

In all these cases, a wavelength of 1 mm was assumed, and the aperture was assumed evenly illuminated.

4.1.1. Properly aligned system.

The principal features are tilt (1,-1) which wasn't entirely taken out in the ray-tracing setup, coma (3,-1), and astigmatism (2,2). All are negligible.

```
Aber. power loss: 0.012958
  0   0 -0.00308
  1  -1 -0.03568   1  0.00000
  2  -2  0.00000   0 -0.00350   2  0.00402
  3  -3  0.00004  -1 -0.00658   1  0.00000   3 -0.00000
  4  -4 -0.00000  -2  0.00000   0  0.00025   2 -0.00007   4 -0.00002
  5  -5  0.00007  -3  0.00014  -1  0.00040   1  0.00000   3  0.00000   5  0.00000
  6  -6 -0.00000  -4 -0.00000  -2  0.00000   0  0.00012   2  0.00001   4  0.00002   6 -0.00003
  0   0  0.00037
  1  -1  0.01257   1  0.00000
  2  -2  0.00000   0  0.00016   2  0.00011
  3  -3  0.00000  -1  0.00021   1  0.00000   3  0.00000
  4  -4  0.00000  -2  0.00000   0  0.00000   2  0.00000   4  0.00000
  5  -5  0.00000  -3  0.00000  -1  0.00000   1  0.00000   3  0.00000   5  0.00000
  6  -6  0.00000  -4  0.00000  -2  0.00000   0  0.00000   2  0.00000   4  0.00000   6  0.00000
```

4.1.2. Defocus of secondary

In this case, the secondary was moved back 0.07 inch, appropriate for focusing on the test transmitter. Tilt and wavefront curvature taken out. Coma and astigmatism remain at more or less their negligible levels, but this time a small amount of spherical (4,0) appears. The power loss is about 1.6%.

```
Aber. power loss: 0.036746
  0   0 -0.06159
  1  -1 -0.04340   1  0.00000
  2  -2 -0.00000   0 -0.01199   2  0.00386
  3  -3 -0.00362  -1 -0.00739   1 -0.00000   3  0.00000
  4  -4  0.00000  -2  0.00000   0  0.04524   2 -0.00007   4  0.00053
  5  -5  0.00043  -3  0.00063  -1  0.00048   1  0.00000   3 -0.00000   5  0.00000
  6  -6 -0.00000  -4  0.00000  -2 -0.00000   0 -0.00572   2  0.00013   4  0.00063   6  0.00012
  0   0  0.14974
  1  -1  0.01859   1  0.00000
  2  -2  0.00000   0  0.00189   2  0.00010
  3  -3  0.00006  -1  0.00027   1  0.00000   3  0.00000
  4  -4  0.00000  -2  0.00000   0  0.01616   2  0.00000   4  0.00000
  5  -5  0.00000  -3  0.00000  -1  0.00000   1  0.00000   3  0.00000   5  0.00000
  6  -6  0.00000  -4  0.00000  -2  0.00000   0  0.00018   2  0.00000   4  0.00000   6  0.00000
```

4.1.3. Tilted secondary

It is useful to check this one out, since nutation of the secondary to move off-source has been proposed. If you tilt the secondary about its vertex, not much happens, neither with the aberrations nor with the tilt of the wavefront. A 0.2 degree subreflector tilt results in a 0.036 degree tilt of the wavefront, or change in direction on the sky. Very slight amounts of coma and astigmatism. The tilt was in the *y-z* plane.

```
Aber. power loss: 0.043027
  0   0 -0.01520
  1  -1 -0.00616   1  0.00000
  2  -2  0.00000   0 -0.01097   2  0.01793
  3  -3  0.00003  -1 -0.08920   1  0.00000   3 -0.00000
  4  -4  0.00000  -2  0.00000   0  0.00060   2 -0.00071   4  0.00014
  5  -5  0.00032  -3  0.00056  -1  0.00877   1  0.00000   3 -0.00000   5  0.00000
  6  -6 -0.00000  -4  0.00000  -2  0.00000   0  0.00048   2  0.00002   4  0.00002   6  0.00000
  0   0  0.00913
  1  -1  0.00037   1  0.00000
  2  -2  0.00000   0  0.00158   2  0.00212
  3  -3  0.00000  -1  0.03927   1  0.00000   3  0.00000
  4  -4  0.00000  -2  0.00000   0  0.00000   2  0.00000   4  0.00000
  5  -5  0.00000  -3  0.00000  -1  0.00025   1  0.00000   3  0.00000   5  0.00000
  6  -6  0.00000  -4  0.00000  -2  0.00000   0  0.00000   2  0.00000   4  0.00000   6  0.00000
```

4.1.4. Tilted Primary

A tilted primary is a different matter. A tilt of only 0.03 degree results in a hefty Strehl loss of 7%, mostly in the (3,-1) coma mode. (The tilt was in the *y-z* plane only.)

```
Aber. power loss: 0.069955
  0   0  0.00126
  1  -1 -0.00365   1  0.00000
  2  -2 -0.00000   0 -0.00469   2 -0.00062
  3  -3 -0.00015  -1  0.11962   1  0.00000   3 -0.00000
  4  -4 -0.00000  -2 -0.00000   0  0.00023   2  0.00034   4 -0.00033
  5  -5 -0.00051  -3 -0.00069  -1 -0.01387   1  0.00000   3 -0.00000   5 -0.00000
  6  -6 -0.00000  -4 -0.00000  -2 -0.00000   0 -0.00059   2  0.00001   4 -0.00004   6 -0.00012
  0   0  0.00006
  1  -1  0.00013   1  0.00000
  2  -2  0.00000   0  0.00029   2  0.00000
  3  -3  0.00000  -1  0.07061   1  0.00000   3  0.00000
  4  -4  0.00000  -2  0.00000   0  0.00000   2  0.00000   4  0.00000
  5  -5  0.00000  -3  0.00000  -1  0.00063   1  0.00000   3  0.00000   5  0.00000
  6  -6  0.00000  -4  0.00000  -2  0.00000   0  0.00000   2  0.00000   4  0.00000   6  0.00000
```

4.1.5. Offset Secondary

In this case, the secondary was offset 0.1 inch in the *x*-direction. A considerable amount of coma now appears in the form of a (3,1) mode, accounting for most of the 26% power loss. Clearly, offsets of the secondary are verboten. A suitable tolerance for offsets of the secondary would be perhaps half this, producing only ¼ the loss.

It should perhaps be noted that tilts of the primary and offsets of the secondary are really equivalent, and both must be held to close tolerances.

```
Aber. power loss: 0.261962
  0   0 -0.00957
  1  -1 -0.00668   1 -0.03366
  2  -2 -0.00745   0 -0.00824   2 -0.00096
  3  -3 -0.00018  -1 -0.00681   1  0.23778   3  0.00140
```

```
  4  -4  0.00155  -2  0.00050   0  0.00094   2  0.00065   4  0.00002
  5  -5  0.00009  -3  0.00022  -1  0.00039   1 -0.02554   3  0.00172   5  0.00029
  6  -6 -0.00042  -4 -0.00057  -2 -0.00204   0  0.00001   2 -0.00011   4  0.00004   6 -0.00003
  0   0  0.00361
  1  -1  0.00044   1  0.01118
  2  -2  0.00036   0  0.00089   2  0.00001
  3  -3  0.00000  -1  0.00023   1  0.27902   3  0.00001
  4  -4  0.00001  -2  0.00000   0  0.00001   2  0.00000   4  0.00000
  5  -5  0.00000  -3  0.00000  -1  0.00000   1  0.00215   3  0.00001   5  0.00000
  6  -6  0.00000  -4  0.00000  -2  0.00001   0  0.00000   2  0.00000   4  0.00000   6  0.00000
```

4.2. Holography

Mel Wright, Dick Plambeck, and Douglas Bock have made several holograms of Antenna 4, both at the infinity focus and with a subreflector setting adjusted for the location of the test transmitter. Little if any difference could be found between the two focal settings. The data used here were taken from file `holo/hmap56_4.2.transfocus.45A`. These data, as the name implies, were taken at the transmitter's focus. Wavelength 3mm.

A filter program was constructed so that the amplitudes and phases from the holograph image could be formatted for Zernike analysis, in the same manner the rays were processed.

The Zernike coefficients show a trace amount of coma (3,-1 shows 0.031 with a 0.5% power loss) and traces of astigmatism (2,2 shows 0.013 with 0.1% loss). They will of course scale up as you go up in frequency. The data were weighted by the amplitudes read from the hologram file. Numerous outlier phases had to be deleted from the filter file prior to Zernike processing. (These all seemed to take on the spurious value -0.795775 radian.)

```
Aber. power loss: 0.012299
  0   0 -0.00134
  1  -1 -0.00150   1 -0.00086
  2  -2 -0.00369   0 -0.00145   2  0.01260
  3  -3 -0.00396  -1  0.03107   1  0.00993   3  0.00840
  4  -4  0.00030  -2  0.01038   0 -0.00412   2 -0.00045   4 -0.00166
  5  -5  0.00336  -3 -0.00172  -1 -0.01611   1  0.00269   3  0.00079   5 -0.00055
  6  -6  0.00153  -4 -0.00124  -2 -0.00133   0 -0.00140   2 -0.00125   4  0.00102   6 -0.00039
  0   0  0.00007
  1  -1  0.00002   1  0.00001
  2  -2  0.00009   0  0.00003   2  0.00104
  3  -3  0.00008  -1  0.00476   1  0.00049   3  0.00035
  4  -4  0.00000  -2  0.00043   0  0.00013   2  0.00000   4  0.00001
  5  -5  0.00004  -3  0.00001  -1  0.00085   1  0.00002   3  0.00000   5  0.00000
  6  -6  0.00001  -4  0.00000  -2  0.00001   0  0.00001   2  0.00000   4  0.00000   6  0.00000
```

These results are consistent with those of the program that generated the hologram, which indicated in its history file:

```
HOLOFFT: Phase RMS (radians) before fit:     0.124
HOLOFFT: Number of points in phase fit =   2021
```

```
HOLOFFT: Results of the Phase Fit (Phase(rad)=A+Bx+Cy+D(x*x+y*y)) are:
HOLOFFT: where A,B,C,D (radians) =  0.27629 -0.06674 -0.11283 -0.05766
HOLOFFT: Phase RMS (radians) after fit:     0.105
HOLOFFT: Surface RMS (microns):    29.844
HOLOFFT: Median (microns) is        -3.741
```

A 0.105 radian RMS corresponds to a Strehl loss of 1.1%.

The amplitude and phase profiles are shown in Fig. 1.

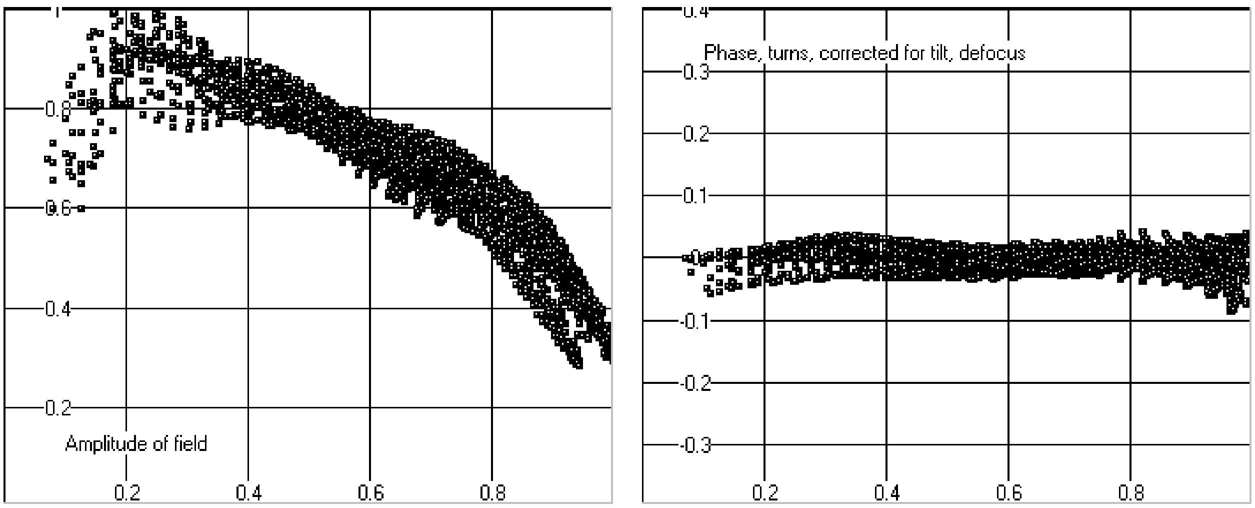

Fig. 1. Amplitude and phase of field on Antenna 4, composite of radial slices.

5. Conclusions

5.1. Alignment Tolerances

Alignment tolerances for the Cassegrain optics may be given as follows:

Condition	Tolerance	Strehl Loss
Feed offset from axis	5 inches	0.1%
Secondary mirror defocus	0.07 inch	1.6%
Secondary mirror tilt	0.1 degree	1.0%
Primary mirror tilt	0.01 deg.	0.7%
Secondary mirror offset	0.02 inch	1.0%

A wavelength of 1mm is assumed. In some cases, a linear interpolation from the ray-trace data was done; this is justified, since the aberration here was mostly coma, and coma scales with the field angle or tilt.

The field of view actually exceeds the conservative estimate of 5 inches radius. One must take account, however, of field curvature (12 inch radius) in locating outlying feeds. The condition on secondary defocus is fairly stringent, but doesn't really constitute a problem for us at 1 mm. One should use the infinity focus, and apply a parabolic phase correction to the data, and thereby avoid the problem. The primary must have its axis accurately aligned, and the secondary must not be offset more than 0.02 inch from the parabola's axis.

5.2. Assessment of Antenna 4

The optics appear to be well-aligned and the panels well-tuned. What looks good at 3 mm, however, will not play so well at 1mm. The (3,-1) coefficient will increase from 0.031 to 0.9, with a corresponding loss of 4.2%. Other losses will go up by factors of 9, as well. However, none of this can explain the rather low efficiencies reported by Dick Plambeck.

The antenna illumination in Fig. 1 suggests possible spillover losses, which of course won't be seen in the holography or ray-tracing. The illumination pattern has high efficiency at 3mm (~80%). This computation ignores spillover. There is an evident need to repeat the holography at 1mm when a transmitter becomes available.

References

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